Construction of Golden Hexagon with golden section in equilateral Triangle

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Abstract

In this article, the golden section is related with equilateral triangle and golden hexagons. Simple construction method for making golden ratio hexagon is presented from equilateral triangle with golden sections on each triangle lengths. From the method, golden sections and golden ratio appears in hexagon for which ratio of two irregular sides of hexagon gives the value of golden ratio phi. The motivation behind this work is: it is important to learn about phi φ which is mathematically more challenged, its application to geometry and mathematics.

Keywords

Golden sections, Golden ratio, Golden Hexagon, Geometry and Graphics

Introduction

Golden ratio is an irrational number that's equal to approximately 1.6180 and is written by Greek letter φ. When we divide a line into two parts such that the whole length is divided by the long part is also equal to the long part divided by the short part. For example,

$$\frac{AB}{AC} = \frac{AC}{CB} = \varphi \cong 1.6180 \tag{1}$$

Geometry and Proof

Consider an equilateral triangle ABC as shown in Fig. 1. Let the points E, D and G on each sides of the triangle such as[2]

$$\frac{CE}{EA} = \frac{BD}{DC} = \frac{AG}{GB} = \varphi \tag{2}$$

Where φ^3 is the golden ratio.

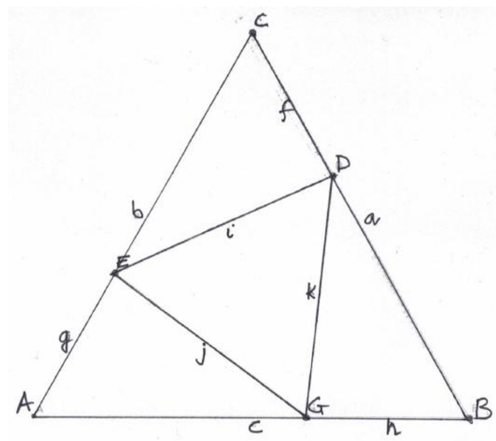

Figure 1: Two equilateral triangle

Construct triangle EDG inside triangle ABC.

In triangle EDG, Obtain points I, j and k in such a way EI =ID, EJ=JG, DK=KG.

Also, draw line passing through vertex C and a point I intersects at line AB at point M, similarly, draw line passing through vertex B and point k which interacts line AC at point **K.** Similarly, also draw line passing through the vertex A and point j which intersects line CD at L. (Fig. 2).

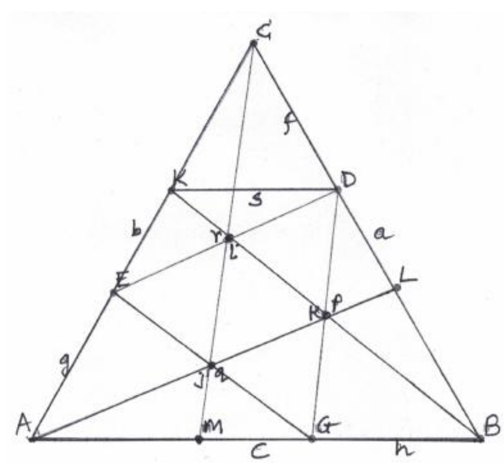

Figure 2: Golden sections and vertices

Construct hexagon KDLGMEK (Fig. 3) for which

$\frac{KD}{KE} = \frac{EM}{MG} = \frac{GL}{LD} = \phi$.

Proof for $\frac{KD}{KE} = \phi$:

By applying cosine rule for triangle CED,

$$ED^2 = CE^2 + CD^2 - 2CE \times CD \times \cos 60^\circ \quad (3)$$

Also, it can be seen from triangle ABC, $\frac{CE}{CD} = \varphi \Rightarrow CE = \varphi CD; CD = \frac{CD}{\varphi}$

Substituting above value into equation (3)

$$ED^2 = (\varphi CD)^2 + CD^2 - 2\varphi CD \times CD \cos 60^\circ$$
$$ED^2 = \varphi^2 CD^2 + CD^2 - 2\varphi CD \times CD\left(\frac{1}{2}\right)$$
$$ED^2 = CD^2\left(\varphi^2 - \varphi + 1\right)$$
$$ED = \sqrt{CD^2\left(\varphi^2 - \varphi + 1\right)}$$

$$ED = CD\sqrt{\left(\varphi^2 - \varphi + 1\right)} \quad (4)$$

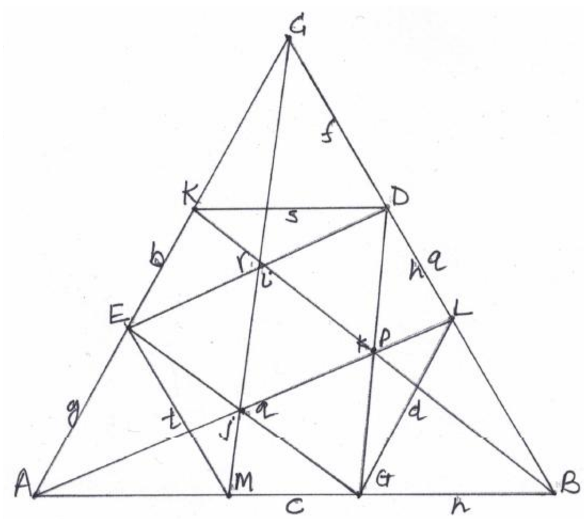

Figure 3: Golden hexagon

Here, Take, CD = KD such as, triangle CKD gives another equilateral triangle for which angle CKD is 60 degree. Since KD is horizontal parallel line to line AB, angle DKE is 120 degree.

Now, applying sine rule for the triangle KDE, knowing length ED as in equation (4), and the angle DKE,

$$\frac{ED}{\sin K} = \frac{KD}{\sin E} = \frac{EK}{\sin D} \tag{5}$$

$$\frac{CD\sqrt{\left(\varphi^2 - \varphi + 1\right)}}{\sin 120} = \frac{KD}{\sin E} = \frac{EK}{\sin D}; \tag{6}$$

Knowing $KD = CD$, substituting in equation (6),

$$\frac{CD\sqrt{\left(\varphi^2 - \varphi + 1\right)}}{\sin 120} = \frac{CD}{\sin E}$$

$$\sin E = \frac{CD \times \sin 120}{CD\sqrt{\left(\varphi^2 - \varphi + 1\right)}} \tag{7}$$

$$\sin E = \frac{\sin 120}{\sqrt{\left(\varphi^2 - \varphi + 1\right)}}$$

$$E = \sin^{-1} \frac{\sin 120}{\sqrt{\left(\varphi^2 - \varphi + 1\right)}} \tag{8}$$

$$D = 180 - E - K$$

$$D = 180 - \sin^{-1} \frac{\sin 120}{\sqrt{\left(\varphi^2 - \varphi + 1\right)}} - 120 \tag{9}$$

From equation (6),

$$\frac{KD}{EK} = \frac{\sin E}{\sin D} =; \frac{\sin\left(\sin^{-1} \frac{\sin 120}{\sqrt{\left(\varphi^2 - \varphi + 1\right)}}\right)}{\sin\left(180 - \sin^{-1} \frac{\sin 120}{\sqrt{\left(\varphi^2 - \varphi + 1\right)}} - 120\right)} = \varphi$$

Hence given proof for, $\frac{KD}{EK} = \varphi$.

Also, it can be observed from the triangle, KC = CD;

$$\mathrm{KC=CD;KE=EC\text{-}KC}$$

$$\frac{\mathrm{EC}}{\mathrm{CD}} = \varphi; EC = \varphi CD$$

$$\mathrm{KE}=\varphi CD\text{-}\mathrm{CD}$$

$$\mathrm{KE=CD}\left(\varphi\text{-}1\right)$$

$$\frac{CD}{KE} = \frac{1}{(\varphi - 1)} = \frac{KC}{KE} = \varphi$$

Conclusion

In this article, golden sections, equilateral triangle and golden hexagon are related with each other. The final geometry gives golden hexagon with irregular adjacent sides. This paper gives fundamental design and construction of golden hexagon. The geometrical proof has been given for the ratio of the two adjacent sides of the golden hexagon which gives the value φ. The significance of the work: there are many interesting and infinite mathematical relationships and oddities in Phi φ that can be explored in more depth.

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