# Resolving Vectors

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**Abstract**

Static or dynamic state of any object is the result of various forces acting on it. To correlate the state of the object mathematically with the forces, one need to analyse opposing forces and complimentary forces by the process of 'Vector Resolution', meaning breaking of a vector in two components perpendicular to each other. One has to then write governing equilibrium or force balancing equations and derive the formulae for the object's motion. Present article emphasizes the very simple fact that there are no sacred directions along which the force vectors should be resolved. The key step is to draw a rectangle with given force vector along its diagonal and orienting the rectangle in such a way that its sides should be parallel (or anti-parallel) with other force vectors acting on the object. This unified method has been illustrated by different examples in the article.

**Keywords**

Vectors, resolution of vectors, classroom teaching

**Introduction**

Any physical quantity that requires magnitude as well as direction for its interpretation is called as a vector. In daily life, often one needs to use judgement to find which way a big carton is to be pushed/pulled/lifted in order to relocate it from one place to another. In cricket, a bowler needs to use a judgement as to how the ball needs to be thrown so that it reaches the batsman in the most complicated manner. Although in both these examples, neither a worker nor a bowler takes a break and calculate the force to be applied or how the ball is to be thrown. The action taken normally depends on a quick strategy which gets developed with experience and practice!

For science and engineering practitioners, it is of utmost importance to understand role of vectorial quantities and how to deal with them. While teaching many analytical topics one often needs to begin with equilibrium equations and derive further. A common


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observation is that students either don't understand how to deal with various force vectors to find resultant or they just accept what is written in the books. If the students cannot derive these conditions from scratch then they end up memorizing the salient steps which leads to 'Volatile Learning'.

Present article discusses concepts taught in classrooms to first or second year undergraduate students. The article will first focus on additions/subtractions of vectors. Then common methods to resolve vectors into two components will be discussed. A few examples discussed in the article represent how a single parallelogram method of vector resolution can be used in solving any problem involving force vectors.

**Discussion**

***Combination of Vectors- Addition & Subtraction:***

Two or more vectors can add with or subtract from each other to give a final resultant vector. However, addition or subtraction of vectors is done in a graphical way. 'Scalar' quantities which do not require direction for their description along with magnitude, can be added or subtracted quickly in mind to get a final estimate. But it is not obvious to guess how a resultant of two given vectors will look like unless you draw them on paper. The operator of addition or subtraction can be simply done by graphical method. For addition of two vectors $\vec{A}$ and $\vec{B}$, tail of vector $\vec{B}$ should be placed on head of vector $\vec{A}$. Now draw a new vector with its tail from tail of vector $\vec{A}$ to head of vector $\vec{B}$. When vector $\vec{B}$ is to be subtracted from vector $\vec{A}$, first reverse the direction of vector $\vec{B}$ and then place tail of inverted vector $\vec{B}$ on head of vector $\vec{A}$. Next to find the resultant vector draw a new vector from tail of vector $\vec{A}$ to head to inverted vector $\vec{B}$. Figure 1 shows an example of the vector addition and subtraction.

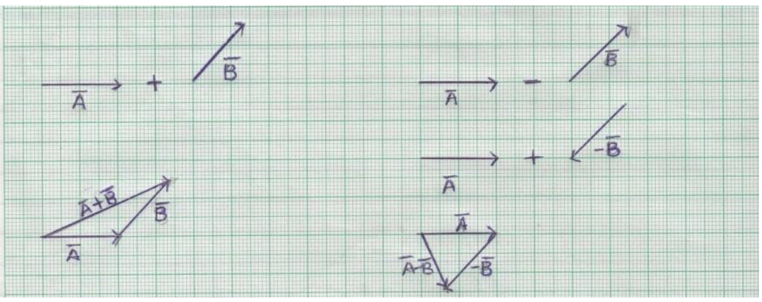
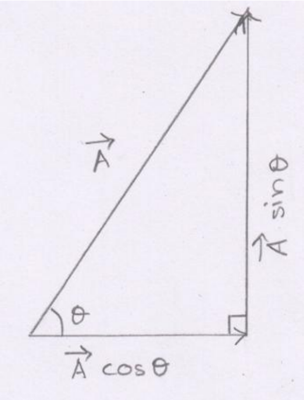


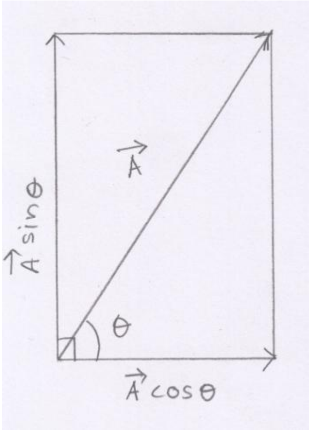

**Figure 1: Addition and Subtraction of vectors**

***Resolution of Vectors:***

Every vector can be represented as addition of two perpendicular vector components. Most of the times, we want to know how a given vector can be written as a sum of horizontal and vertical vector components in the plane containing the vector. There are two methods (1) trigonometry method and (2) parallelogram method as shown below in figure 2.




**(a)**




**(b)**

**Figure 2: (a) Trigonometry method (b) Parallelogram method**

In trigonometry method as shown in figure 2(a), the given vector needs to be the resultant of addition of two vectors perpendicular to each other. In parallelogram method (fig. 2(b)), a rectangular parallelogram is built around the given vector in such a way that the given vector is along the diagonal to the rectangle. The height and width of the rectangle form the two components of the given vector. Now the question is whether resolution of vector means to break the given vector in horizontal and vertical components? The answer is 'NO'. It is very important to realize that for a given vector there are infinite possibilities to draw a rectangle having the given vector along its diagonal (figure 3).

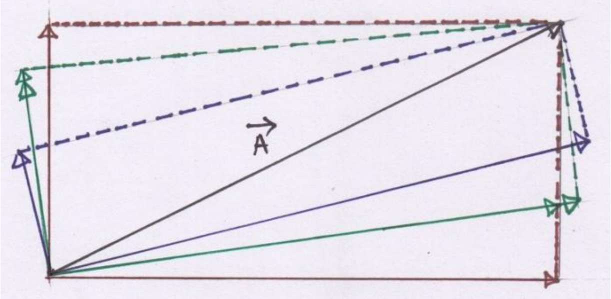
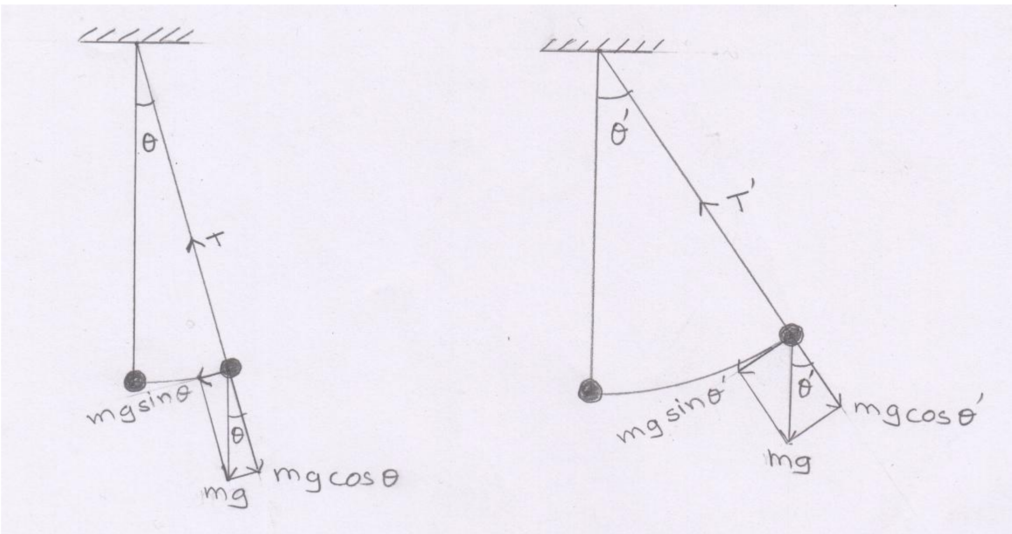



**Figure 3: Parallelogram method showing many possibilities of rectangles for the same diagonal vector**

As shown in figure 3, like red, green, blue, one can draw a number of rectangles with a given vector along the diagonal. So technically, $\vec{A}$ can be written as a resultant of any pair of red, green or blue vectors and not just the horizontal and vertical (red) vectors. Note that the magnitudes and directions of vector components can be different depending upon which rectangle we choose for diagonal formation. So then the question is which way the vectors need to be resolved. The answer is that one should resolve the given vector in those two perpendicular components which help in solving equilibrium conditions in free body diagram. This point is illustrated below by taking four examples.

1. *Oscillation of a simple pendulum*[1]*:*




**(a)** **(b)**

**Figure 4: (a) Pendulum slightly away from equilibrium position (small $\theta$) and (b) Pendulum far away from equilibrium position (large $\theta'$)**

Both the above figures show oscillatory motion of a simple pendulum with free body diagrams. In both (a) and (b), the gravitational force (weight) acts in the pendulum vertically downward direction but we choose to resolve the weight vector along two different perpendicular components in such a way that one of the components is along opposite of one of the tension force T in the string and the other component is responsible to move the pendulum towards the centre.

As shown in figure 4(a) and (b), the weight force is resolved into two perpendicular components such that cos θ component is opposite to the tension force in the string.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However, as the pendulum moves from the central position towards extreme position, the angle the gravitational force makes with the string increases, which leads to increase in the sin θ component.

$$T = mg\cos\theta$$
$$mg\sin\theta = ma$$

*Physical significance of resolved vectors:*

a. Tension in the string tied to an oscillating simple pendulum is not same at all positions of the pendulum. It reduces as the pendulum moves away from the central position.
b. The restoring force acing on the pendulum to bring it back increases as the pendulum moves away from the central position and is maximum when in the extreme position pertaining to maximum value of mg sin θ.

2. *Wooden box sliding on an inclined plane with reasonable friction[1]:*

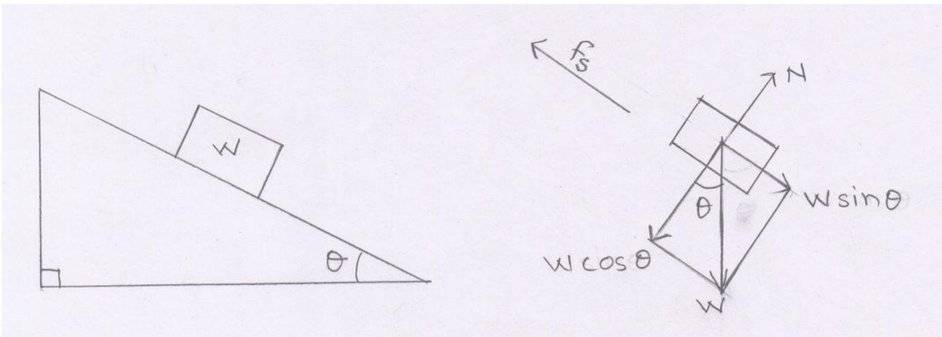


**Figure 5: Wooden box on inclined plane and its free body diagram**

Since the block is moving along the inclined surface, the normal reaction has to balance the W cosθ component of the gravitational force. Since the block is sliding down wards, the frictional force which acts parallel to the surface and opposite to the motion does not balance the W sinθ component of the gravitational force.

$$W\sin\theta - f_s = ma$$
$$W\sin\theta - \mu N = ma$$
$$N = W\cos\theta$$

*Physical significance of resolved vectors:*

a. If the normal reaction is more than $W\cos\theta$ component the block will lift above the surface while sliding.
b. If the coefficient of friction is sufficiently large, then it is possible that the block does not slide at all. The sliding can be stopped even when the angle of inclination becomes less.

3. *Stress on an inclined section[2]:*

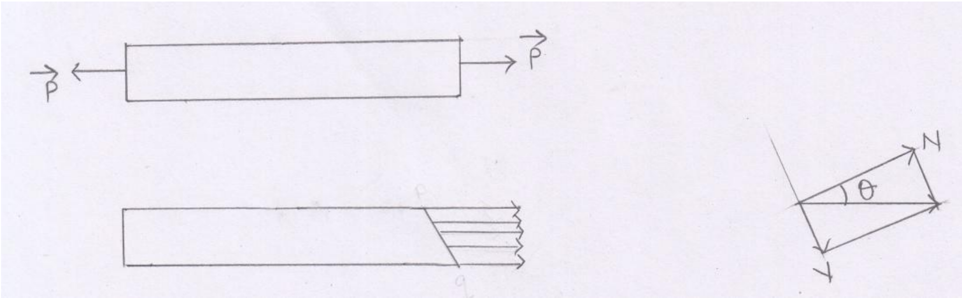


**Figure 6: Normal force (N) and shear force (V) acting on an intermediate inclined section of a prismatic bar under tensile load**

Consider a homogeneous, prismatic bar with rectangular cross-section being subjected to tensile load. Imagine what kinds of stresses are present at an inclined surface. For equilibrium of the bar, the applied load P at the left end and the developed force due to stress at the right has to balance each other. This developed stress gets uniformly distributed over any cut section surface along the horizontal line. So the equivalent load P acting on the inclined surface pq can be resolved in two vectors N and V. Force N acts along the perpendicular direction to the inclined surface and force V parallel to the surface pq.

*Physical significance of resolved vectors:*

Even though the rectangular bar is subjected to tensile load along the axis of the bar, on an intermediately cut inclined section, there are two kinds of stresses present, normal stress $\sigma_\theta$ and shear stress $\tau_\theta$. Note that the perpendicular component N is responsible for the tensile/normal stress $\sigma_\theta$ and parallel component V is responsible for the shear stress $\tau_\theta$.


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## 4. *Forces acting on soap bubbles[3]:*

A soap bubble can be modelled in the form of a spherical shell. As shown in figure 8, there are three types of forces acting on the bubble. (i) Force due to outside air acting radially inwards (ii) Force due to air inside acting radially outwards and (iii) force due to surface tension along horizontal axis. In order to find resultant of force due to outside air, one has to resolve each force vector along two components, one horizontal $P_1 \Delta S \cos\theta$ and one vertical $P_1 \Delta S \sin\theta$. When considered over the entire spherical surface, all the vertical components get cancelled out and only the horizontal components survive. The horizontal components $P_1 \Delta S \cos\theta$ due to all area elements of the spherical shell need to be added up. But $\Delta S \cos\theta$ is projection of area element of the hemisphere on the circular vertical plane passing through the diameter of the sphere. So when all the area elements of the hemispherical surface are projected, one gets the circular plane itself. So the force acting on the bubble due to outside air is equal to $P_1(\pi r^2)$, where $r$ is the radius of the sphere. The same trick can be used to determine force acting on the shell due to air inside the shell.

*Physical significance of resolution of vectors:*

The resolved horizontal component acting on all area elements needs to be taken throughout the hemispherical surface and later integrated. Or the effective area is to be considered as the circular cross-sectional area of the spherical shell and the pressure acting on the area is to be taken as $P_1$.

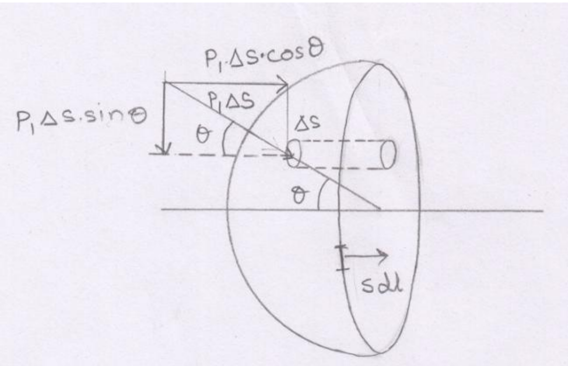


**Figure 7: A hemispherical shell of a soap bubble showing force acting on it due to outside air and surface tension**


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## Suggestions

A take home message from this article for the students is to understand the basic steps to resolve a given vector in two perpendicular vector components by parallelogram method. The two components or the sides of the rectangle are to be chosen in such a way that they are either parallel or anti parallel to other vectors in the free body diagram. Identify the angle the vector makes with the sides of the rectangle and find out the magnitudes of the vector components. These component values can then be used in equilibrium or balancing equations to get final answer. These simple steps can also serve as teaching pedagogy for teaching relevant topics.

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